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On concrete characterization problem of universal graphic automata

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Graphic automaton is an automaton $A = (X, S, \delta)$, for which the set of states is equipped with the structure of a graph $G = (X, \rho)$ preserved by transition functions of the automaton. Graphic automaton $\mathrm{Atm}(G) = (G, \mathrm{End}\ G, \delta)$, where $\delta(x, \varphi) = \varphi(x)$ for $x \in X,\ \varphi \in \mathrm{End}\ G$, is a universally attracted object in the category of automata [1]. For this reason the automaton $\mathrm{Atm}(G)$ is called universal graphic automaton over the graph G.

Let $G = (X, \rho)$ be a graph. An edge $(x, y) \in \rho$ is called proper if $(y, x) \notin \rho$. A graph is called quasi-acyclic if all its proper edges don't belong to any cycle. In this paper we investigate the following concrete characterization problem of these automata: under which conditions for an automaton $A = (X, S, \delta)$ it is possible to construct on the set X a structure of reflexive quasi-acyclic graph $G = (X, \rho)$, such that the equality $A = \mathrm{Atm}(G)$ holds.

Let $A = (X, S, \delta)$ be an automaton without equal acting input symbols, where S is a semigroup of transformations of X. For $x, y, u, v \in X$ the symbol $\begin{pmatrix} x & y \\ u & v \end{pmatrix}$ means that there exists $s \in S$ satisfying $\delta_s(X) = \{u, v\},\ \delta_s(x) = u,\ \delta_s(y) = v$.

For an automaton $A = (X, S, \delta)$ the canonical binary relations Q, R on the set X are defined by the formulas:

$$Q = \left\{ (x, y) \in X^2 | \begin{pmatrix} x & y \\ x & y \end{pmatrix} \wedge \neg \begin{pmatrix} x & y \\ y & x \end{pmatrix} \right\},\ R = \left\{ (x, y) \in X^2 | \, (\forall u, v \in X, u \neq v) \begin{pmatrix} u & v \\ x & y \end{pmatrix} \right\}.$$

For an automaton $A = (X, S, \delta)$ the semigroup S is called QR-closed if for every transformation f of the set X, from the condition that for any pair $(x, y) \in Q \cup R$ there is an input symbol $s \in S$, such that δ_s coincides with f on the set $\{x, y\}$, it follows that $f = \delta_a$ for some $a \in S$.

Theorem. *Let $A = (X, S, \delta)$ be an automaton without equal acting input symbols. Then A is universal graphic automaton $\mathrm{Atm}(G)$ for some reflexive quasi-acyclic graph $G = (X, \rho)$ if and only if the input symbol semigroup S is QR-closed and its canonical relations Q, R satisfy the following conditions:*

(1) $(x, x) \in R$ *for all* $x \in X$;

(2) $(x, y) \in Q \wedge (u, v) \in Q \Longrightarrow \left(\begin{pmatrix} x & y \\ u & v \end{pmatrix} \Longleftrightarrow \neg \begin{pmatrix} x & y \\ v & u \end{pmatrix} \right)$;

(3) $(x, y) \in Q \wedge \begin{pmatrix} x & y \\ u & v \end{pmatrix} \Longrightarrow \left(\begin{pmatrix} x & y \\ v & u \end{pmatrix} \wedge (u, v) \in R \quad \vee \quad \neg \begin{pmatrix} x & y \\ v & u \end{pmatrix} \wedge (u, v) \in Q \right)$.

References

1) Plotkin B. I., Greenglaz L. Ja., Gvaramija A. A. Algebraic structures in automata and databases theory. *Singapore; River Edge*, NJ : World Scientific, 1992.